KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PRACTICE PAPER 1

JUNE 2024

MARKS: 150

TIME: 3 hours

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-2)(5+x)=0$ (2)

1.1.2 $3x^2-2x-6=0$ (correct to TWO decimal places) (4)

1.1.3 $2\sqrt{x+6}+2=x$ (4)

1.1.4 $x^2<-2x+15$ (4)

1.1.5 $2^{x+2}-3.2^{x-1}=80$ (5)

1.2 Solve for x and y simultaneously:

$3^{x+y}=27$ and $x^2+y^2=17$ (6)

1.3 Show that $2.5^n-5^{n+1}+5^{n+2}$ is even for all integer values of n. (3)

1.4 Determine the values of x and y if: $\frac{3^{y+1}}{32}=\sqrt{96^x}$ (4)

[32]

QUESTION 2

2.1 Given a quadratic number pattern: $-120;\ -99;\ -80;\ -63;\ldots$

2.1.1 Write down the next TWO terms of the pattern. (2)

2.1.2 Determine the n^{th} term of the number pattern in the form $T_n=an^2+bn+c$. (4)

2.1.3 What value must be added to T_n for the sequence to have only one value of n for which $T_n=0$? (4)

[10]

QUESTION 3

3.1 Given a finite arithmetic series: $9+14+19+\ldots+124$.

3.1.1 Determine the general term of this series in the form $T_n=dn+c$. (2)

3.1.2 Write the series in sigma notation. (3)

3.2 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first n terms is $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$. (4)

[9]

QUESTION 4

4.1 Given: $5;\ 10;\ 20;\ \ldots$ a geometric sequence.

4.1.1 Determine the n^{th} term. (1)

4.1.2 Calculate the sum of the first 18 terms. (2)

4.2 The first and second terms of a geometric series is given as: $2x-4;\ 4x^2-16;\ \ldots$
Determine the value(s) of x for which the series will converge. (4)

4.3 A convergent geometric series has a first term of 2 and $r=\frac{1}{\sqrt{2}}$.
Calculate sum to infinity divide by sum of two terms. (3)
[10]

QUESTION 5

The line $y=x+1$ and $y=-x-7$ are the axes of symmetry of the function $f(x)=\frac{-2}{x+p}+q$.

5.1 Show that $p=4$ and $q=-3$. (3)

5.2 Calculate the x-intercept of f. (2)

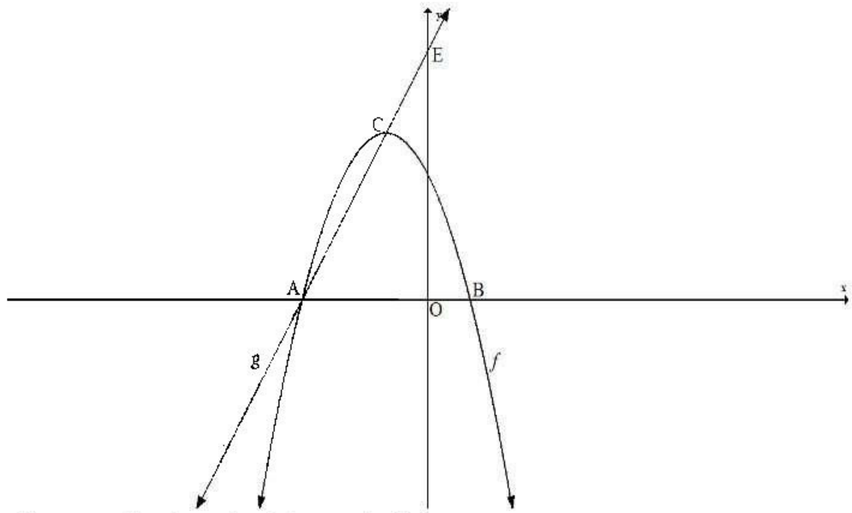
5.3 Sketch the graph of f. Clearly label ALL intercepts with the axes and the asymptotes. (4)

5.4 Write down the equation of the vertical asymptote of the graph of h. if $h(x)=f(x+5)$. (2)

5.5 Determine the values of x for which $f(x)>0$. (2)
[13]

QUESTION 6

The sketch below shows the graphd of $f(x)=-x^2-2x+3$ and $g(x)=mx+q$.
Graph f has x-intercepts at A and $B(1\ ;\ 0)$ and a turning point at C.
The straight line g. passing through A and C, cuts the y-axis at E.



6.1 Write down the coordinates of y-intercept of f. (1)

6.2 Show that the coordinates of C are $(-1\ ;\ 4)$. (3)

6.3 Write down the coordinates of A. (1)

6.4 Calculate the length of EC. (6)

6.5 Determine the value of k if $h(x) = 2x + k$ is a tangent to the graph of f. (5)

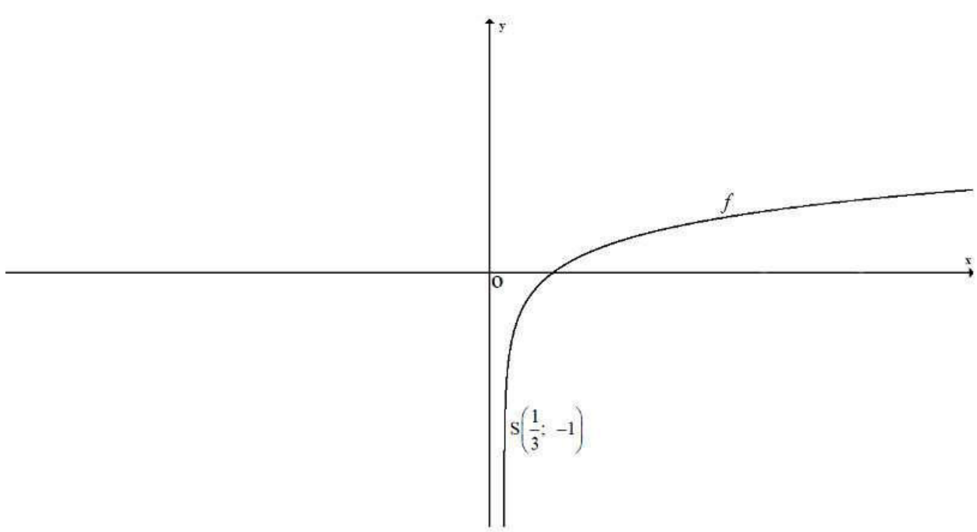
6.6 Determine the equation of g^{-1}, the inverse of g, in the form $y = ...$ (2)

6.7 For which value(s) of x is $g(x) \geq g^{-1}(x)$? (2)

[20]

QUESTION 7

Given: $f(x) = \log_a x$ where $a > 0$. $S\left(\frac{1}{3};\ -1\right)$ is a point on the graph of f.



7.1 Prove that $a = 3$. (2)

7.2 Write down the equation of h, the inverse of f, in the form $y = ...$ (2)

7.3 If $g(x) = -f(x)$, determine the equation of g. (1)

7.4 Write down the domain of g. (1)

7.5 Determine the values of x for which $f(x) \geq -3$. (3)

[9]

QUESTION 8

8.1 Determine $f'(x)$ from FIRST PRINCIPLES if $f(x) = 3x^2 - 2$. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = 6x^3 - 5x^2 + 13x - 7$ (3)

8.2.2 $D_x\left[8x^2 + \frac{2}{x^3}\right]$ (3)

8.2.3 Given $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the equation of the tangent to the curve at the point $x = 2$. (4)

[15]

QUESTION 9

9.1 Given: $f(x) = 2x^3 - 5x^2 - 4x + 3$.

9.1.1 Calculate the coordinates of the x-intercept. (4)

9.1.2 Write down the coordinate of the y-intercept. (1)

9.1.3 Determine the x-coordinates of the turning points of f. (4)

9.1.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axis. (3)

9.2 Determine the values of x for which:

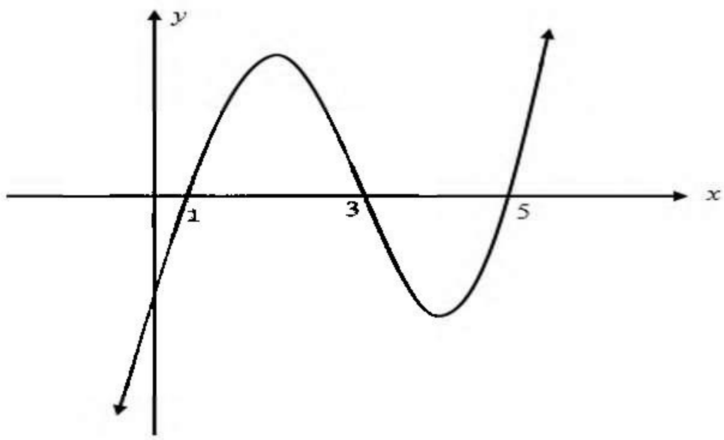
9.2.1 f is concave up. (3)

9.2.2 f is strictly increasing. (2)

[17]

QUESTION 10

The sketch below shows the graph of $h(x) = x^3 - bx^2 + cx - d$.
The coordinates of the x-intercepts of h are (1 ; 0), (3 ; 0) and (5 ; 0).



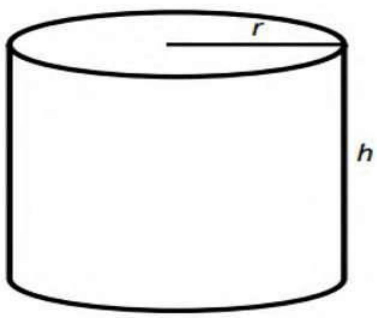
10.1 Determine the values of b, c and d.. (5)

10.2 Determine the values of x for which $f(x) \geq 0$. (3)

[8]

QUESTION 11

A $225\ \text{cm}^3$ cylindrical piece 0f wood with height, h, and radius, r, is shown below.



11.1 Determine the height of the wood in terms of the radius r. (2)

11.2 Show that the surface area of the wood is $A = 2\pi r^2 + \frac{450}{r}$. (2)

11.3 Calculate the radius of the wood (in cm), if the surface area of the wood has to be as small as possible. (3)

[7]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area}\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

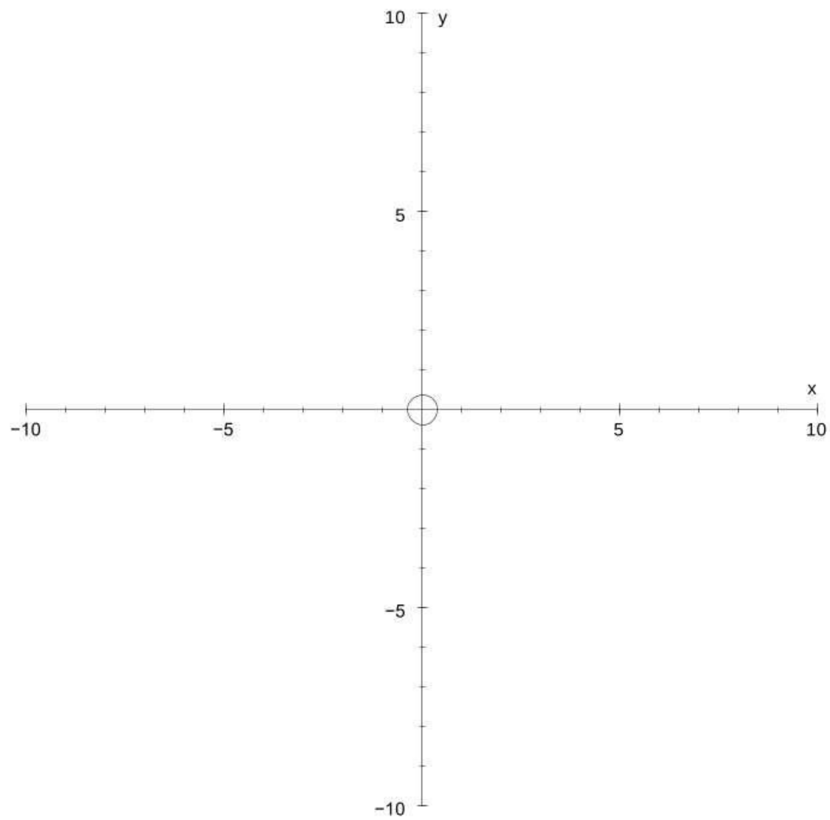
$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

Surname and Names :…………………………….

Question 5.3



Question 9.1.4